Realization of Dirac point with double cones in optics

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The Dirac point (DP) with a double-cone structure for optical fields can be realized in optically homogenous media. The condition for the realization of DP in optical systems is the varying of refractive index from negative to zero and then to positive. Our analysis verify that, similar to electrons in graphene, the light field near DP possesses of the pseudodiffusive property obeying the $1/L$ scaling law, where L is the propagation distance inside the media. © 2009 Optical Society of America

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Recent discovery of graphene [1] refreshes attention to the two-dimensional (2D) massless Dirac equation. In graphene, the conduction and valence bands touch each other at the Dirac point (DP) with a double-cone structure. Near the DP, the dispersion is linear with two branches. Owing to these facts, graphene is not only interesting because of its electronic properties, such as frequency-dependent conductivity [2], but it is also interesting because it is an excellent candidate for experimental demonstrations of quantum relativistic effects, such as Zitterbewegung [3], which are still never directly observed for electrons. In this Letter, we perform a theoretical investigation on realizing the DP in homogenous materials (HMs) for light fields (LFs), thus providing a direct optical analog of graphene. Compare with solids, optical systems offer a clean and easily controlled way to test theoretical predictions. The experimental test in electronic systems is usually hindered by the difficulty to maintain system homogeneity, while for optical systems, no such difficulty exists. Therefore establishing the optical analog of graphene would open up the possibility to study condensed matter analogies in an optical way.

It is well known that Maxwell's equations for LFs reduce to the Helmholtz equation, which could be written as $\wp E_z(x,y,\omega)+k^2(\omega)E_z(x,y,\omega)=0$, with a wavenumber $k(\omega)$ and $\wp=(\partial^2/\partial x^2+\partial^2/\partial y^2)$ in a 2D case for an HM when an LF is polarized in the z direction. Following with the Dirac's idea [4], the operator $\wp$ can be expanded into a matrix form: $\wp=(\sigma_x\partial/\partial x+\sigma_y\partial/\partial y)(\sigma_x\partial/\partial x+\sigma_y\partial/\partial y)$, where σ_x and σ_y are two Pauli matrices. Therefore the Helmholtz equation is written as the Dirac equation,

$$\begin{pmatrix} 0 & -i(\partial/\partial x - i\partial/\partial y) \\ -i(\partial/\partial x + i\partial/\partial y) & 0 \end{pmatrix}\Psi = k(\omega)\Psi, \quad (1)$$

where $\Psi=\binom{E_{z1}(x,y,\omega)}{E_{z2}(x,y,\omega)}$ and $E_{z1,z2}(x,y,\omega)$ are two eigenfunctions of the electric fields with the same $k(\omega)$. In most media, $k(\omega)$ is always positive definite. It means one could not have the DP with a double-cone structure. However, this should not prevent us from realizing the DP in particular optical systems. In photonic crystals (PCs), the DP for the Bloch states [5–7] is found from the similarity of the photonic bands of the 2D PCs with the electronic bands of solids. Several novel optical transport properties near DP have been shown in [6–8], such as conical diffraction [6], a "pseudodiffusive" scaling [7], and the photon's Zitterbewegung [8]. Naturally, it would be of great interest to find out the condition to have the Dirac dispersion for the LF in HMs.

Usually, $k(\omega)$ can be expanded as $k(\omega)=k(\omega_D)+(\omega-\omega_D)/v_D+\beta(\omega-\omega_D)^2+\cdots$ at $\omega_D>0$, with group velocity $v_D=(\mathrm{d}\omega/\mathrm{d}k)|_{\omega=\omega_D}$. If $k(\omega_D)=0$ and the quadratic- and higher-order terms could be neglected, then we would have a linear dispersion

$$k(\omega)=(\omega-\omega_D)/v_D. \quad (2)$$

For media satisfying Eq. (2), $k(\omega)$ varies from negative to zero and then to positive, and so does the refractive index with zero at ω_D. We call such media as the negative-zero–positive-index (NZPI) media. Substituting Eq. (2) into Eq. (1), we have

$$\begin{bmatrix} 0 & -iv_D(\partial/\partial x - i\partial/\partial y) \\ -iv_D(\partial/\partial x + i\partial/\partial y) & 0 \end{bmatrix}\Psi = (\omega-\omega_D)\Psi. \quad (3)$$

Equation (3) is the massless Dirac equation of LFs in HMs, which is similar to that of electrons in graphene [9]. Therefore, for NZPI media [with Eq. (2)], we have the DP with a double-cone structure. Although the condition of Eq. (2) is simple, to our knowledge no one has noticed it in optical systems before. Close to ω_D, owing to $k^2=k_x^2+k_y^2\to 0$, k_x becomes an imaginary number for real k_y; thus the fields along the x direction between the interval L have the following relation: $t(L,k_y)=E(L)/E(0)=\exp(-|k_y|L)$. Then the total energy transmittance is

$$T_{\mathrm{All}} = \int_{-\infty}^{\infty} |t(L,k_y)|^2 \mathrm{d}k_y = 1/L, \quad (4)$$

which tells us that the propagation of LF at ω_D exhibits the $1/L$ scaling, a main characteristic of diffusion phenomenon.

Now the question is how to find such a medium satisfying Eq. (2). Fortunately, the manmade metamaterials (MMs) with small absorption [10–14] have provided a chance to meet the requirement. Some researchers have already demonstrated a type of NZPI MM [11]. For simplicity, we choose the Drude model [12] for the permittivity and permeability of the MMs, $\varepsilon_1(\omega)=1-\omega_{ep}^2/(\omega^2+i\gamma\omega)$, and $\mu_1(\omega)=1-\omega_{\mathrm{mp}}^2/(\omega^2+i\gamma\omega)$, where $\omega_{\mathrm{ep,mp}}$ are the controllable electronic- and magnetic-plasma frequencies [10,13,14], and γ is related to the absorption with $\gamma \ll \omega_{\mathrm{ep,mp}}$. When $\omega_{\mathrm{ep}}=\omega_{\mathrm{mp}}=\omega_D$ and $\gamma=0$, both ε_1 and μ_1 are zero at ω_D, and Eq. (2) is valid near ω_D.

First let us verify the $1/L$ scaling law in a semi-infinite (SI) NZPI MM; see Fig. 1(a). Such a structure may reduce but cannot completely eliminate the non-ideal interface effect ($x=0$). As a simple proof, we assume ε_1 and μ_1 are real. The transmission coefficient at $x=L$ is $t(k_y,\omega)=\alpha\exp[ik_{x1}L]$, where $\alpha=2(q_0q_m)^{1/2}/(q_0+q_m)$ is determined only by the boundary condition, $q_m=k_{x1}/(\mu_1k_0)$, $q_0=(k_0^2-k_y^2)^{1/2}/k_0$ for $k_0>k_y$ and otherwise $q_0=i(k_y^2-k_0^2)^{1/2}/k_0$, where $k_0=\omega/c$ and k_{x1} is the x-component wavenumber in the MM. Near ω_D, we have $t(k_y,\omega)=\alpha\exp[-|k_y|L]$. Usually, α depends on k_y (the interface effect). For large L, the function $\exp[-|k_y|L]$ decreases quickly with the increasing of k_y. In this sense we assume that α is independent of k_y (regarded as an ideal interface). Then the total transmittance is $T_{\mathrm{All}}=\int_{-\infty}^{\infty}|t(k_y,\omega)|^2\mathrm{d}k_y=|\alpha|^2/L$, which is different from Eq. (4) by only a value α owing to the interface. Therefore the light transport near ω_D is proportional to $1/L$ inside the NZPI MM.

Figure 1(b) plots the transmitted spectrum of light at $L=1000$ mm inside the SI MM [Fig. 1(a)]. We see that both the upper and lower passbands touch at $\omega_D/2\pi=10$ GHz, and nearby the dispersion is linear.

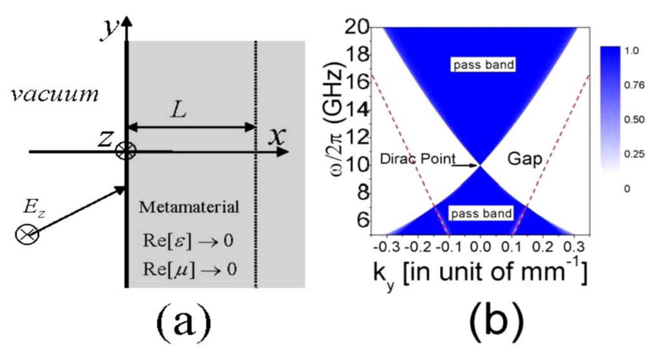

Fig. 1. (Color online) (a) SI system and (b) transmitted spectral distribution at position L. Dashed lines denote the light cone, and dark gray and white areas denote the large transmission (passbands) and the prohibition of light (gaps), respectively. The medium has $\varepsilon_1(\omega)=\mu_1(\omega)=1-\omega_D^2/(\omega^2+i\gamma\omega)$ with $\omega_D=2\pi\times10$ GHz and $\gamma=10^{-5}$ GHz.

As L becomes large, the touch at ω_D is an ideal point. Note that $t(k_y,\omega)$ at DP is close to one even if the MM has a small absorption.

For demonstrating the diffusion near the DP, we use the numerical method [15] to show the spatial evolution of a normally incident Gaussian beam inside a semi-infinite structure, see Figs. 2(a)–2(c). The arrows denote the energy flow $\vec{S}$ (Poynting vector). The incident angular spectral amplitude is $E(k_y,0)=W/\sqrt{2}\exp[-W^2k_y^2/4]$ with a half-width W. Within the upper ($\omega/2\pi=13$ GHz) and lower passbands (8 GHz), the propagation effects are clearly seen in Figs. 2(a) and 2(c). At the DP (10 GHz), the field is a diffusive radiation in Fig. 2(b). Here the total electric field before the interface ($x<0$) is a superposition field of the incident and reflected fields.

For demonstrating the $1/L$ scaling law near DP, a characteristic quantity $\xi=S_rL$ is defined to describe light transport inside the medium, where $S_r\equiv S(x,y=0)/S_0$ is a relative energy flow along the x axis and $S_0\equiv S(x=0,y=0)$ depends on the coupling strength. The dependence of ξ on L is plotted in Fig. 3. Near the DP (10 GHz, 10.2 GHz), ξ tends to be independent of L, i.e., the $1/L$ scaling law. Away from the DP, ξ has a linear dependence of L (i.e., a constant S_r except small absorption). For a comparison, a dotted

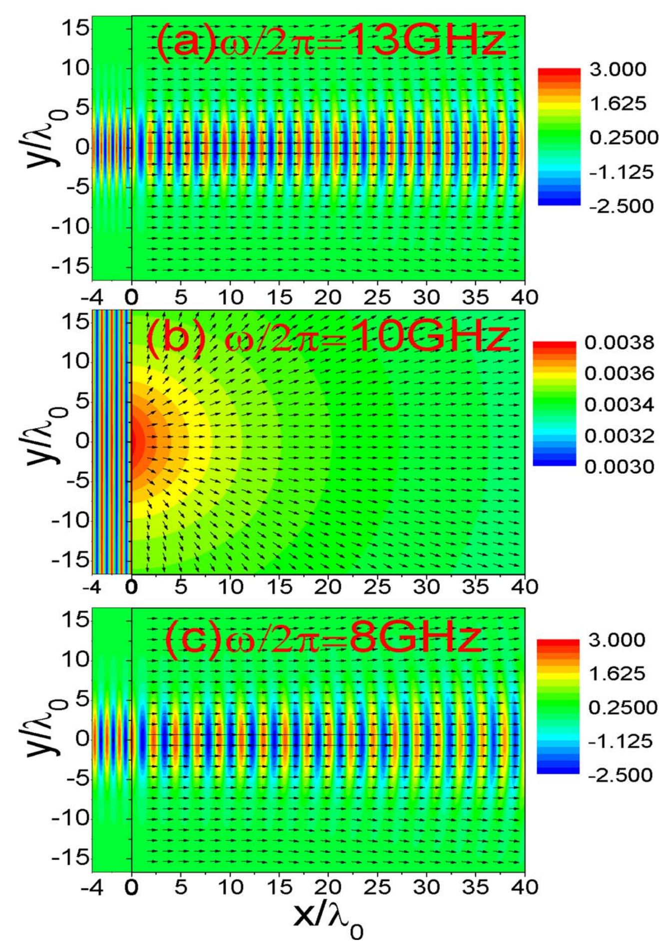

Fig. 2. (Color online) Evolutions of total electric fields for a Gaussian beam through the interface at frequencies with $W=5\lambda_0$ (λ_0 corresponds to ω_D). Other parameters are the same as in Fig. 1 except for $\gamma=10^{-4}$ GHz.

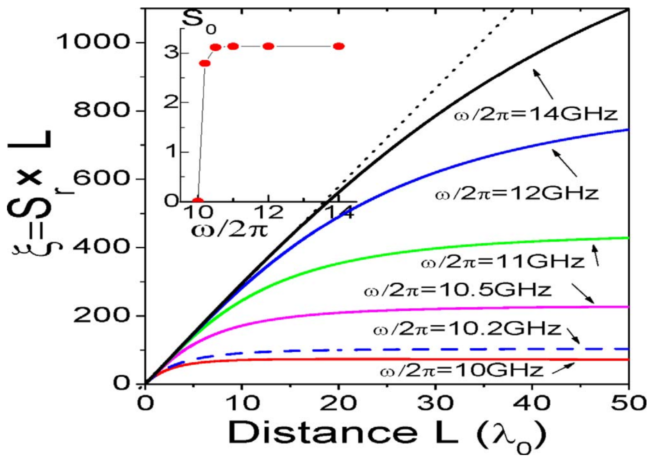

Fig. 3. (Color online) Dependence of ξ on L inside an SI structure under different ω. The inset shows the change of S_0 with ω. Other parameters are the same as in Fig. 2.

line denotes the case for ω within a transparent passband. Figure 4 shows the change of ξ as a function of x inside a realistic MM slab. For a finite thickness d, ξ initially increases and then gradually decays to match the boundary at $x=d$ (the cross points in Fig. 4). For large d, ξ gradually tends to the SI case ($d \to \infty$). Therefore the light transport also obeys the $1/L$ scaling law in a sufficient thick slab.

In a summary, we present the condition for the realization of DP physics in optical systems for the first time to our knowledge. We show that the DP with a double-cone structure exists in HMs as long as Eq. (2)

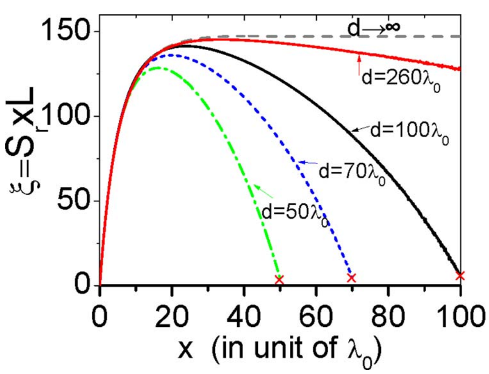

Fig. 4. (Color online) Change of ξ as a function of x inside different slabs, with $\omega=\omega_D$ and $W=10\lambda_0$. Other parameters are the same as in Fig. 2

is valid. The existing NZPI MM is a good candidate. Our analyses reveal that the LF near the DP becomes a diffusive field, and its energy transport obeys the $1/L$ scaling law. In our calculations the absorption is small, so the coupling strength is low near the DP (see inset in Fig. 3). Determining how to enhance the coupling strength of the LF at interface near the DP needs further investigation for its potential applications.

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